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Technical Notes

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Amiet's Theory in Spanwise-Varying Flow Conditions

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Nomenclature

b	=	semichord
c_0	=	sound speed
d	=	semispan
K_y	=	radiating spanwise aerodynamic wave number ($\omega y/c_0\sigma$)
k_x	=	chordwise aerodynamic wave number (ω/U)
k_y	=	spanwise aerodynamic wave number
$\mathcal{L}$	=	aeroacoustic transfer function
M	=	freestream Mach number
n	=	number of airfoil strips
$T.I.$	=	turbulence intensity
U	=	freestream velocity
$\mathbf{x}$	=	observer coordinates (x, y, z)
β	=	compressibility parameter ($\sqrt{1-M^2}$)
ρ_0	=	density
σ	=	far-field corrected distance [$\sqrt{x^2+\beta^2(y^2+z^2)}$]
Φ_{ww}	=	two-dimensional wave number turbulence spectrum
ω	=	circular frequency

Introduction

DEVELOPMENT of aerodynamic theories started in the mid-1920s [1,2] with models dealing with airfoils of infinite span, zero angle of attack, and without camber, in uniform parallel incompressible flows. Upstream flow nonhomogeneities are assumed to be frozen and convected at mean flow velocity. Considering only small-amplitude disturbances, the unsteady aerodynamical problem can be linearized with respect to the steady mean flow.

In 1938, von Kármán and Sears [3] proposed a general approach to compute the lift for any small-amplitude motion of a two-dimensional airfoil in an incompressible flow based on the concept of circulation theory. This approach recovers results obtained 10 years before by Theodorsen [4] on the lift of a sinusoidally oscillating airfoil and by Küssner [5] on the fluctuating lift of a two-dimensional airfoil subjected to a step change in the upwash incoming velocity. Based on this result, in 1941 Sears [6] obtained a simple analytical expression for the fluctuating lift induced by a vortical frozen sinusoidal gust, parallel to the airfoil span and impinging on a fixed thin airfoil in an incompressible flow. Filotas [7] extended Sears's result to an oblique sinusoidal gust.

Later on, Adamczyck [8] proposed a compressible model to compute the airfoil lift response induced by an oblique gust impacting on an infinite span airfoil, including camber effects, using the Wierner–Hopf technique. The method is based on a successive leading- and trailing-edge corrections procedure from an initial solution, first described by Landahl [9]. Amiet [10] obtained similar results with the Schwarzschild method [11] in the simplified case of a profile without camber. Amiet [12] related the dipole repartition (pressure distribution) on the profile to the acoustic radiation in the far field with the help of radiating dipole formulation in a convected free field. Moreau et al. [13] extended this theory to take into account subcritical gusts (subsonic relative phase speed with respect to the moving fluid) through the aeroacoustic transfer function, with these gusts contributing to the total noise radiated in case of a finite aspect ratio.

The main restrictions of these theories concern the assumed uniform upstream flow conditions along the airfoil span. In several industrial applications, as in wind turbines, fans, helicopter rotors, or airfoil in jets [14], the properties of the flow (velocity, turbulence intensity, integral length scale of turbulence) are not constant along the span, which does not allow using such theories. Rozenberg et al. [15] first attempted to treat spanwise-varying conditions in case of trailing-edge noise by cutting the complete airfoil in strips, each having their own flow conditions, and the overall noise radiated being the summation of the noise emitted by each one of the airfoil strips. This Note proposes to highlight the limitations of this method for flow interaction noise (leading-edge noise) and to suggest a procedure to improve the method.

Amiet's Theory and Strip Method

An airfoil of chord $2b$ and span $2d$ is placed in a turbulent fluid with a mean flow velocity U in the axial (chordwise) direction. The origin of the coordinates system is at the center of the airfoil and the observer is placed in the far field. The x and y axes are the chordwise and spanwise directions, respectively. The turbulence is assumed to be frozen and represented in terms of its spectral wave number components, k_x and k_y. The airfoil is assumed to be a flat plate of zero thickness at zero angle of attack, and linearized theory is considered so that the wave number associated with the z direction does not enter into account. Amiet [12] proposed an expression for the far-field acoustic power spectral density (PSD) in terms of the turbulence energy spectrum of the upstream flow interacting with the airfoil (Φ_{ww}) and of the airfoil response function to an incoming gust ($\mathcal{L}$), at the listener position $\mathbf{x} = (x, y, z)$:

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$$S_{PP}(\mathbf{x},\omega) = \left(\frac{\omega z \rho_0 b}{c_0 \sigma^2}\right)^2 U d\pi \int_{-\infty}^{\infty} \frac{\sin^2[d(k_y - K_y)]}{\pi d(k_y - K_y)^2} |\mathcal{L}(\mathbf{x}, k_x, k_y)|^2 \Phi_{ww}(k_x, k_y) \mathrm{d}k_y \tag{1}$$

This formulation can be used normally without any further assumptions on the size of the airfoil. In the present study, the von Kármán model is considered for the two-dimensional wave number turbulent energy spectrum, the two relevant parameters of this model being the rms of the squared velocity $\bar{u}^2$ and the turbulence length scale Λ. If the parameter $Mk_x d$ is large, both Φ_{ww} and $\mathcal{L}$ become nearly independent of k_y allowing them to be taken outside the integral of Eq. (1), yielding in [12]

$$S_{PP}(\mathbf{x},\omega) \approx \left(\frac{\omega z \rho_0 b}{c_0 \sigma^2}\right)^2 U d\pi |\mathcal{L}(\mathbf{x}, k_x, K_y)|^2 \Phi_{ww}(k_x, K_y) \tag{2}$$

To take into account spanwise-varying flow conditions along the airfoil, an airfoil discretization in strips each having their own impacting flow conditions could be applied. Because no condition on the airfoil spanwise dimension is required for the use of formulation (1), the latter could then be applied independently of the strip size. The overall sound radiated by the complete airfoil becomes the summation of the noise emitted by each strip. This procedure is applied to the following example. On one hand, let us consider a large span airfoil ($d = 20C$, $C = 0.041$ m) in uniform flow conditions for which formulations (1) and (2) can be applied without any difference. The flow conditions are $c_0 = 340$ m/s, $\rho_0 = 1.225$ kg/m^3, $U = 13.2$ m/s, $T.I. = 0.2$, and $\Lambda = 0.005$ m. The listener position is placed in the far-field at position $\mathbf{x} = (0, 0, 1000C)$. The corresponding acoustic PSD at the listener position is shown in Fig. 1. On the other hand, let us consider the same airfoil, cut in several strips on which formulation (1) is applied. The flow conditions are then the same for each strip as for the complete airfoil. The difference between the complete airfoil and the stripped airfoil can then only be due to the cut itself, as illustrated in Fig. 1 for 2, 8, 32, 64, and 128 airfoil strips. It is shown that the cut is mainly influencing the low frequencies, and the higher the number of strips, the higher the influence of the cut. Furthermore, increasing the number of strips is damping the high frequency secondary lobes. The effect of the cut could be explained by the fact that the total radiated noise is assumed the sum of the strips acting as uncorrelated sources. In principle, strips can be uncorrelated only if their spanwise extent exceeds the spanwise correlation length of the equivalent sources. Intuitively as the wave number k_y is low enough, adjacent strips cannot be uncorrelated and the phase variations which occur over the actual span cannot be reproduced. This link is still under investigation. However, this small example highlights discrepancies appearing with the airfoil strip method, even for a poor discretization along the span ($\approx 0.3C$ with 128 strips) that could not be sufficient for most of the industrial applications.

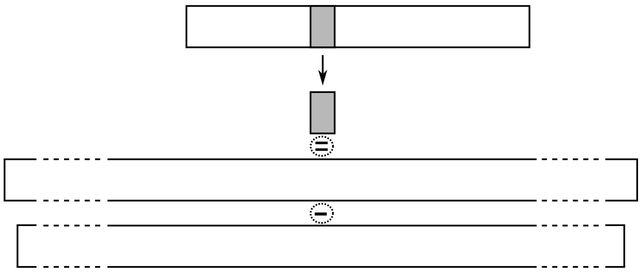

Fig. 2 Illustration of the inverse strip method based on a combination of large span airfoils.

Inverse Strip Method

Because small strips cannot capture properly large aerodynamic wavelengths, the proposed solution is to generate small span strips with a combination of large span airfoils. This procedure is illustrated in Fig. 2. To compute the noise from a small span airfoil strip, a large span airfoil is considered from which is subtracted the same large span airfoil truncated by the considered small span strip. This method allows the use of formulation (2) which, by definition, can capture small frequencies if the aspect ratio is high enough, and which is computationally less expensive compared to formulation (1), avoiding the integral resolution. The inverse method is compared to the direct method in Fig. 3, for the same example as in the previous section with 128 strips. The airfoil semispan used for the strip reconstitution in this method is fixed to $d = 20C$. The inverse method is shown to be able to correctly reproduce the noise radiated within all the frequency range without any discrepancies. This method has been tested on a combination of flow parameters described in Table 1 for which similar results have been obtained. The number of strips has been considered up to 400 strips, the result

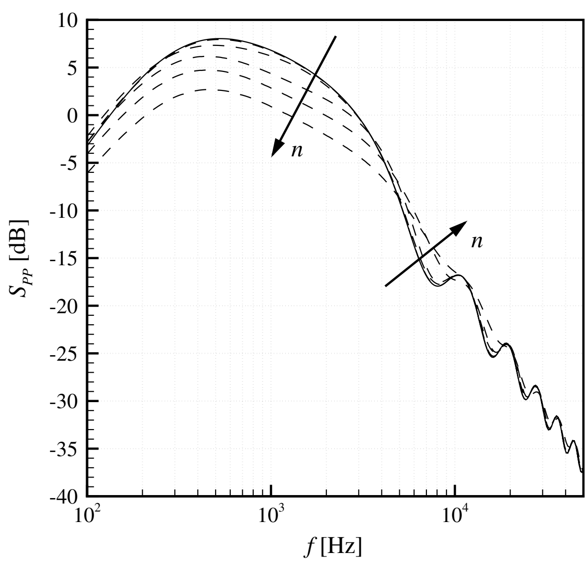

Fig. 1 Far-field acoustic PSD of an airfoil of semispan $d = 20C$ (solid line) compared to sound radiated by the same airfoil cut in strips ($n = 2$, 8, 32, 64, and 128) with the direct method (dashed lines) at the receiver position $\mathbf{x} = (0, 0, 1000C)$ ($c_0 = 340$ m/s, $\rho_0 = 1.225$ kg/m^3, $U = 13.2$ m/s, T.I. = 0.2, and $\Lambda = 0.005$ m).

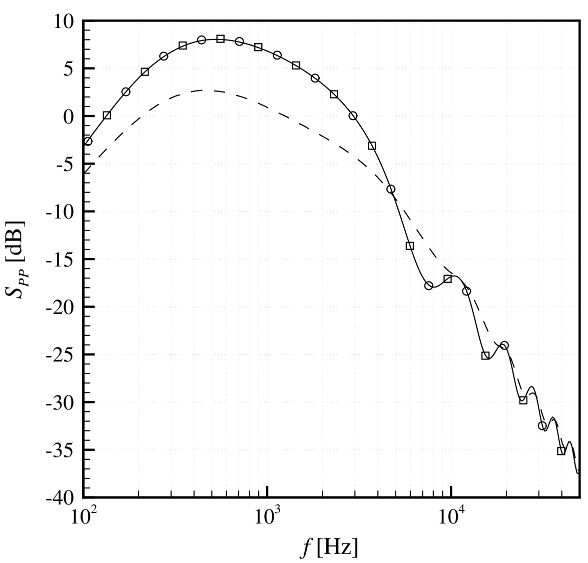

Fig. 3 Far-field acoustic PSD of an airfoil of semispan $d = 20C$ (solid line) compared to sound radiated by the same airfoil cut with the direct method (dashed line) and the inverse strip method (circles) in 128 strips, and the same airfoil cut with the inverse strip method in 400 strips (squares), at the receiver position $\mathbf{x} = (0, 0, 1000C)$ ($c_0 = 340$ m/s, $\rho_0 = 1.225$ kg/m^3, $U = 13.2$ m/s, $T.I. = 0.2$, and $\Lambda = 0.005$ m).

Table 1 Tested parameters

Parameter	Min	Max	Step
Λ, m	0.001	0.01	0.001
$T.I.$	0.1	0.1	0.5
U, m/s	1.0	1.0	30.0

being shown in Fig. 3, and corresponding to a strip size of $0.01C$, allowing now an acceptable discretization of the incoming flow along the span for the industrial applications mentioned above.

Conclusions

Based on Amiet's theory, this paper reviews the possible application of the strip method to predict the noise radiated by airfoils in spanwise-varying upstream flow conditions. This method consists of cutting the airfoil in strips each having its own upstream flow conditions and to sum the resulting individual emitted noise to obtain the total radiated noise in the far field. It appears that this classical method, even with a small number of strips, is not able to correctly compute the far-field noise. A new inverse strip method is proposed to address this problem by using a combination of large span airfoils. This inverse method has been tested for several flow parameters and has shown its potential to correctly reproduce the radiated noise and its possible consideration for future applications.

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